ON THE USE OF THE GENERAL BANDWIDTH METHOD TO IDENTIFY DAMPING IN MDOF SYSTEMS

Dinh Huong Vu[1,*]

[1]*Institute of Techniques for Special Engineering, Le Quy Don Technical University, Hanoi, Vietnam*

Abstract

The general bandwidth (GeB) method is one of the methods used for viscous damping identification of structures. In this method, approximate and exact formulas have been studied to estimate the damping ratio from frequency response function (FRF) of the single degree-of-freedom (SDOF) system. However, the applicability of these formulas to estimate the damping in multi degree-of-freedom (MDOF) systems has not been analyzed in detail. This article studies the influence of modal parameters on the accuracy of the GeB method used to identify the viscous damping ratios in MDOF systems. The results compared with the formulas based on the half-power bandwidth (HPB) method show that the GeB method has good efficiency and a wide identification range.

Keywords: *Damping identification; general bandwidth method; half-power bandwidth method; MDOF systems; frequency response function.*

1. Introduction

Identifying the vibration characteristics of structures (modal frequencies, modal shapes and damping ratios) is an important content in engineering mechanics. In particular, identifying the structural damping parameters is a complicated problem and is being researched by many scientists. Damping identification methods are divided into many different groups, such as: Damping identification for each mode separately, damping identification for many modes, identification in the time domain and identification in the frequency domain.

The classical HPB method is known as a simple and widely used method to identify damping in experimental research on structural vibrations. Accordingly, the viscous damping ratio estimated by the classical formula from experimental data measuring displacement or acceleration FRFs, is equal to half the frequency bandwidth corresponding to the FRF signal power level being half the maximum power [1, 2].

The classical formula for damping estimation is an approximate formula and is often used when the structure has a very small damping ratio. Therefore, many formulas have been developed to improve the accuracy of the HPB method. In 2008, Yin [3, 4] proposed an accurate formula to estimate the damping ratio of a single degree-of-freedom

[*] Email: vudinhhuong@lqdtu.edu.vn
DOI: 10.56651/lqdtu.jst.v7.n01.834.sce

(DOF) system based on the squared FRF curve. With the assumptions that viscous damping is proportional to stiffness, mass and is constant, Olmos [5] used the HPB method to identify damping for MDOF systems. Afterwards, Wang [6] used the HPB method and proposed a 3rd order formula to estimate the viscous damping ratio of the SDOF system. Papagiannopoulos et al. [7] developed a 3rd order formulas based on the HPB method to estimate the damping ratios for MDOF systems. Wang [8, 9] compared the 3rd order formula with the classical formula to identify the damping ratios of the 2-DOF system. In general, the more separated the natural frequencies of structures, the smaller the damping estimate error, and when damping level decreases, the accuracy of the HPB method increases. In 2014, Wu [10] proposed a formula to determine damping ratio by ignoring the 6th order infinitesimal quantity. This formula has quite high accuracy when estimating the damping ratio of the SDOF system. However, the limitation of the Wu's formula and also the general limitation of HPB methods is that its application range is still only within the damping range ($0 \div 0.383$). Furthermore, these damping estimation formulas are all based on a SDOF system and when applied to MDOF systems, they need to be considered specifically.

In 2015, Vu Dinh Huong et al. [11] used the GeB method and proposed an accurate formula to estimate the viscous damping ratio from the displacement spectrum of the SDOF system. This formula is generalized to determine the damping ratio according to the power factor r instead of 2 as in the HPB method. The analysis results on the SDOF system show that the GeB method identifies damping more accurately than the HPB method and can estimate high damping level. Then, the author also used the GeB method to identify the hysteresis damping from the acceleration FRF [12]. Later, Wu [13] also analyzed the displacement spectrum to provide a formula to estimate the damping ratio according to the power ratio, which is similar to the formula in [11] and then the author investigated the influence of the power ratio on the accuracy of the damping estimation formula in the MDOF system.

The GeB method is a new damping estimation method that has been researched recently. The possibility of extending this method to estimate damping in MDOF systems has not yet been studied in detail. In the MDOF systems, there are many parameters that affect the results of damping identification.

This article studies the influence of modal parameters on the accuracy of the GeB method to identify the viscous damping ratios in MDOF systems. The results compared with the formulas based on the HPB method show that the efficiency and identification range of GeB method.

2. Damping identification methods

This section presents a summary of the damping estimation formulas using the methods mentioned above.

2.1. Damping estimation formulas by the half-power bandwidth method

The classical formula for damping estimation based on the displacement FRF of SDOF system has the following form [1, 2]:

$$\xi = \frac{\omega_b - \omega_a}{2\omega_r} = \frac{b}{2} \tag{1}$$

where b is the frequency bandwidth; ω_a and ω_b are two frequencies at the amplitudes equal to $1/\sqrt{2}$ the maximum amplitude (at resonance frequency ω_r).

By continuously expanding the square root expressions, in 2011, Wang [6] proposed a cubic equation to determine the viscous damping ratio from the displacement FRF as follows:

$$4\xi^3 + 2\xi = b \tag{2}$$

Using the HPB method and ignoring 6th order infinitesimals of the damping ratio, Wu [10] proposed the formula to determine the ξ:

$$\xi = \frac{b}{2\sqrt{1+b^2}} \tag{3}$$

Wu's formula gives the best damping estimation in the group of formulas based on the HPB method. However, the Wu-HPB method also has certain limitations when compared with the GeB method (details are presented in the section 4).

2.2. General bandwidth method

In the HPB method, the frequency band width is determined from two frequencies ω_a and ω_b with the FRF amplitudes equal to $1/\sqrt{2}$ the maximum amplitude of the FRF. When the structure has a high damping level or has close natural frequencies, the frequencies ω_a and/or ω_b are outside the resonance peak (Fig. 1). To overcome this limitation, the GeB method is proposed, with two frequencies $\bar{\omega}_1$ and $\bar{\omega}_2$ corresponding FRF amplitude equal to $1/\sqrt{r}$ the maximum amplitude, in which the coefficient r is called the power factor and has any value greater than 1.

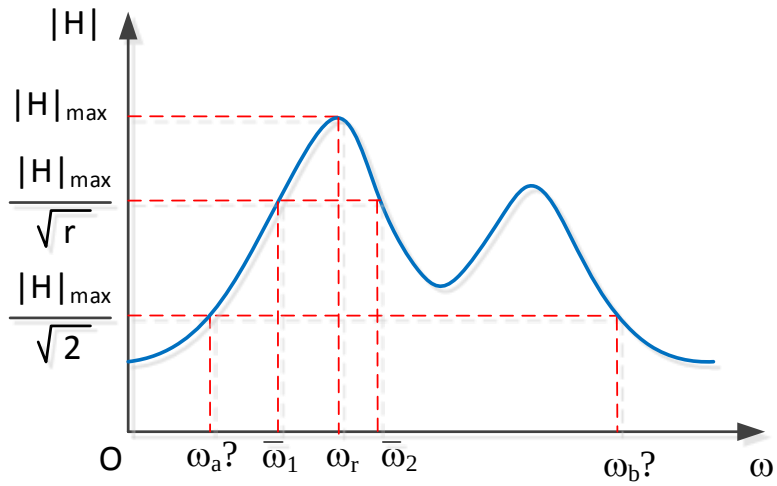

Fig. 1. GeB method with the power factor.

Based on the analysis of the displacement spectrum and general power factor, Vu Dinh Huong [11] has proposed the formula to estimate the viscous damping ratio:

$$\xi = \sqrt{\frac{1}{2} - \sqrt{\frac{r-1}{4(r-1)+4b_r^2 - b_r^4}}} \tag{4}$$

where b is the general frequency bandwidth, r is general power factor,

$$b_r = \frac{\bar{\omega}_2 - \bar{\omega}_1}{\omega_r} \tag{5}$$

with $\bar{\omega}_1$ and $\bar{\omega}_2$ are two frequencies at the amplitudes equal to $1/\sqrt{r}$ the maximum amplitude.

When identifying the viscous damping of the SDOF system, the errors of formula (4) based on the GeB method and the formulas based on HPB method are presented in detail in [11]. A similar analysis of the MDOF system will be conducted in the following sections.

3. Simulated FRF measurement data and estimate damping of MDOF system using GeB method

The experimental FRF measurement data is the ratio between the output parameter $x(t)$ and the input force $f(t)$ in the frequency domain [2, 14]:

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \tag{6}$$

where $X(\omega)$ and $F(\omega)$ are the Fourier transforms of $x(t)$ and $f(t)$, respectively.

Simulated FRF measurement data of MDOF systems is given by the following equation [2, 8]:

$$H(\omega)=\sum_{n=1}^{N}\frac{a_n}{1+2i\xi_n(\omega/\omega_n)-(\omega/\omega_n)^2} \tag{7}$$

where ω_n, ξ_n are the natural frequency and the damping ratio of the n^{th} mode, respectively; a_n is the amplitude coefficient of the n^{th} mode; N is the number of degrees of freedom of the system.

The displacement FRF amplitude graph according to (7) has the form shown in Fig. 2.

The damping identification process is performed for each mode by separating the frequency domain corresponding to each mode (the frequency domain around the resonance peak and can extend to the minimum point (Fig. 2)). Then, determine the general frequency bandwidth from maximum power factor r_{max}:

$$r_{max}=\frac{H_{max}^2}{H_{min}^2} \tag{8}$$

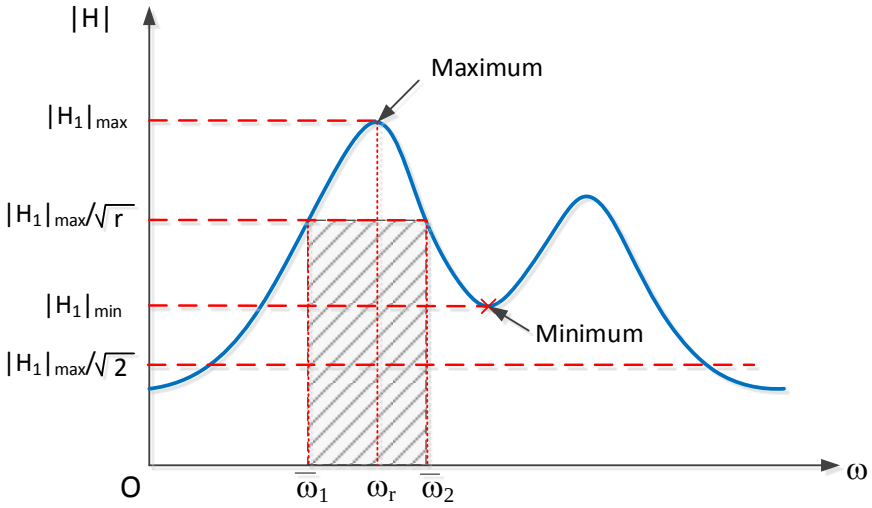

Fig. 2. Identify the damping of the first mode using GeB method.

Consider the following cases:

- Case 1: If $r_{max} \geq 2$, both HPB and GeB methods can estimate the damping ratio. Then, choose $r = 5/4$, the formula to estimate the damping by the GeB method is as follows (9):

$$\xi = \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{\frac{1}{1+4b_r^2 - b_r^4}}} \tag{9}$$

- Case 2: If $2 > r_{max} \geq 5/4$, then the HPB method cannot estimate the damping parameter because the half-power bandwidth does not exist. On the contrary, the GeB method can still estimate the damping ratio according to formula (9) with $r = 5/4$.

- Case 3: If $r_{max} < 5/4$, choose $r = r_{max}$, the GeB method can still identify the damping ratio using formula (4).

4. Comparison of the accuracy of damping identification methods in MDOF systems

This section evaluates the damping estimation error in the MDOF system by the GeB method, then the results are compared with the HPB method using the classical method and Wu-HPB method (Wu's formula).

Considering the 2-DOFs system, the displacement FRF function is simulated according to the following formula:

$$H(\omega) = \frac{a_1}{1-(\omega/\omega_1)^2 + 2i\xi_1(\omega/\omega_1)} + \frac{a_2}{1-(\omega/\omega_2)^2 + 2i\xi_2(\omega/\omega_2)} \tag{10}$$

From (10) it can be seen that the displacement FRF function $H(\omega)$ depends on many parameters that affect the accuracy of damping identification methods: damping level (damping ratios ξ_1 and ξ_2), natural frequencies ω_1 and ω_2, amplitude coefficients a_1 and a_2. Some results of investigating the influence of the parameters on the damping estimation error by the identification methods are presented in the sections below.

4.1. Effect of the damping level on the damping estimation error

For varying damping levels (assuming damping ratios $\xi_1 = \xi_2$), investigate the error in estimating the damping ratios of the first and second modes using the above formulas (with natural frequency ratio $\omega_2/\omega_1 = 4$). The results are given in Fig. 3 and Fig. 4 with amplitude coefficients $a_2 = a_1$ and $a_2 = -a_1$, respectively.

Figure 3 and Fig. 4 show that the damping estimation error using the GeB method is significantly smaller than the classical and Wu's formulas based on HPB method. The range of damping identification using the GeB method is also larger than that one determined by the above formulas. When the damping level increases, the error variation of the GeB method is smaller than that of the HPB method. That shows the effectiveness of the GeB method for structures with high damping levels. For example, in Fig. 3a, at the damping level of 0.05, the damping estimation errors of the first mode by the classical

method (classical form of HPB method), Wu-HPB method (Wu's formula), and the GeB method are not too different: 4.19%, 3.63%, and 2.44%, respectively. But at the damping level of 0.2, the corresponding errors are too different: 71.51%, 41.43%, and 24.82% in that order.

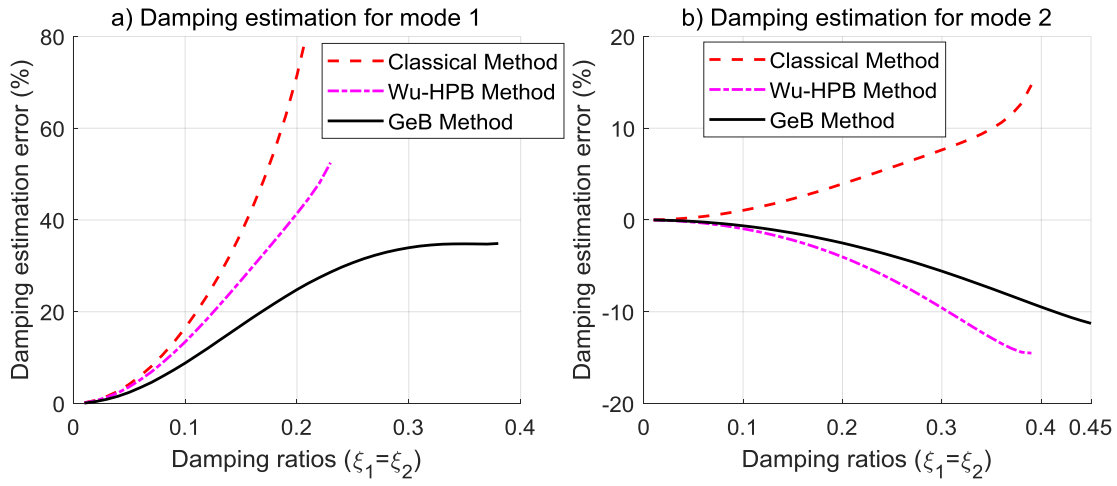

Fig. 3. Dependence of damping estimation error on the damping level (with $a_2 = a_1$).

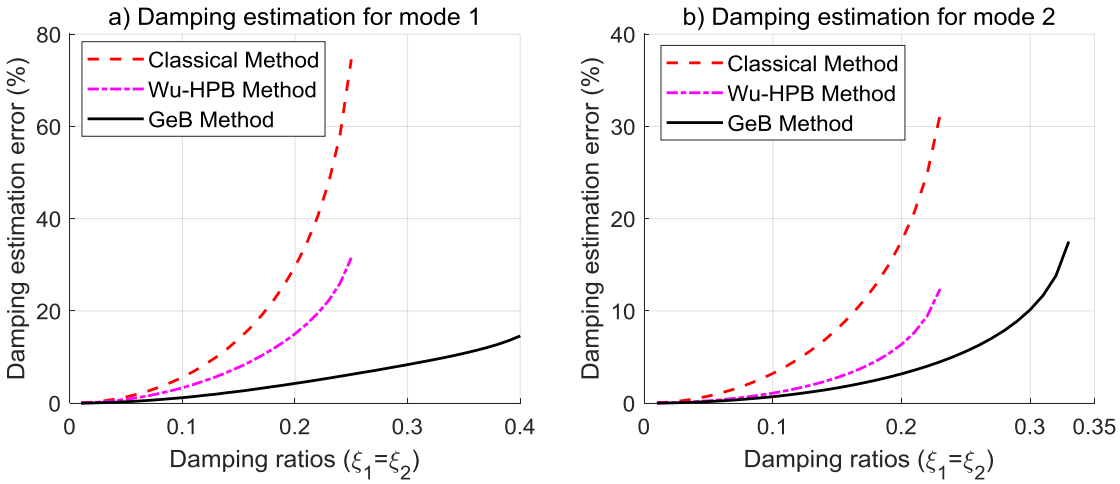

Fig. 4. Dependence of damping estimation error on the damping level (with $a_2 = -a_1$).

Specifically, the classical and Wu-HPB methods can only estimate damping level of the first mode within a range of less than 0.23 (Fig. 3a) and similarly which of the second mode within a range of less than 0.38. (Fig. 3b). Meanwhile, the GeB method can estimate the damping ratio of the first and second mode up to 0.38 (Fig. 3a) and 0.45 (Fig. 3b) respectively. The results in Fig. 4 also show similar observations, the range of damping identification using the GeB method is also larger than that one using the HPB method.

Table 1 and Table 2 present more detailed results on the damping estimation error according to the assumed damping ratios (with amplitude coefficients $a_2 = a_1$). The damping level of a certain mode greatly affects the damping estimation error of itself. The higher the resistance level, the larger the error. The results also show that the damping level of the second mode does not have much impact on the damping estimation error of the first mode and vice versa.

Table 1. Damping estimation error of the first mode (%)

Method	**Classical Method**			**Wu - HPB Method**			**GeB Method**		
Assumed damping	$\xi_1 = 0.05$	$\xi_1 = 0.1$	$\xi_1 = 0.3$	$\xi_1 = 0.05$	$\xi_1 = 0.1$	$\xi_1 = 0.3$	$\xi_1 = 0.05$	$\xi_1 = 0.1$	$\xi_1 = 0.3$
$\xi_2 = 0.05$	4.19	16.75	-	3.63	13.69	-	2.44	8.98	38.56
$\xi_2 = 0.1$	4.15	16.46	-	3.59	13.42	-	2.41	8.81	37.5
$\xi_2 = 0.3$	3.96	15.43	-	3.40	12.47	-	2.28	8.19	33.96

Table 2. Damping estimation error of the second mode (%)

Method	**Classical Method**			**Wu - HPB Method**			**GeB Method**		
Assumed damping	$\xi_1 = 0.1$	$\xi_1 = 0.2$	$\xi_1 = 0.4$	$\xi_1 = 0.1$	$\xi_1 = 0.2$	$\xi_1 = 0.4$	$\xi_1 = 0.1$	$\xi_1 = 0.2$	$\xi_1 = 0.4$
$\xi_2 = 0.1$	1.05	3.7	1.16	-0.95	-4.21	-0.85	-0.63	-2.63	-0.57
$\xi_2 = 0.2$	1.07	3.94	4.53	-0.93	-4.02	-3.56	-0.62	-2.51	-2.24
$\xi_2 = 0.4$	2.69	5.35	-	-20.65	-19.44	-	-12.33	-11.51	-9.48

For the first mode (Table 1), when the damping level is small ($\xi_1 = 0.05$), all three formulas have quite accurate identification results (the error is less than 5%). However, when the damping level increases ($\xi_1 = 0.1$), the formulas based on the HPB method give a large identification error of over 10% (about 13 ÷ 16%), while the error of the GeB method is only about 8 ÷ 9%. Especially, when the damping level is large ($\xi_1 = 0.3$), the HPB method cannot determine the damping parameters while the GeB method can still estimate the damping ratio, but the error is quite large (over 30%).

At the same damping level, the GeB method also estimates the damping ratio of the second mode more accurately than the HPB method (Table 2). Some negative errors indicate that the estimated damping parameter is smaller than the assumed damping level. The damping estimation error of the second mode is significantly smaller than that of the first mode, which can be explained by the fact that the damping ratio of the first mode is more sensitive to influences from other mode than the second mode.

4.2. Effect of the natural frequency ratio ω_2/ω_1 on the damping estimation error

For varying natural frequency ratio ω_2/ω_1, investigate the error in estimating the damping ratios of the first and second modes using the above formulas (with amplitude coefficients $a_1 = - a_2$). The results are given in Fig. 5 and Fig. 6 with damping ratios $\xi_1 = \xi_2 = 0.10$ and $\xi_1 = \xi_2 = 0.30$, respectively.

When the structure has separate frequencies (and damping is small - Fig. 5), the damping estimation errors by the methods are also small. The errors increase as the natural frequency ratio decreases. When the structure has two close natural frequencies (ω_2/ω_1 is close to 1), the errors increase rapidly and identification methods are no longer accurate. Even in this case, the GeB method still gives better results than the HPB method.

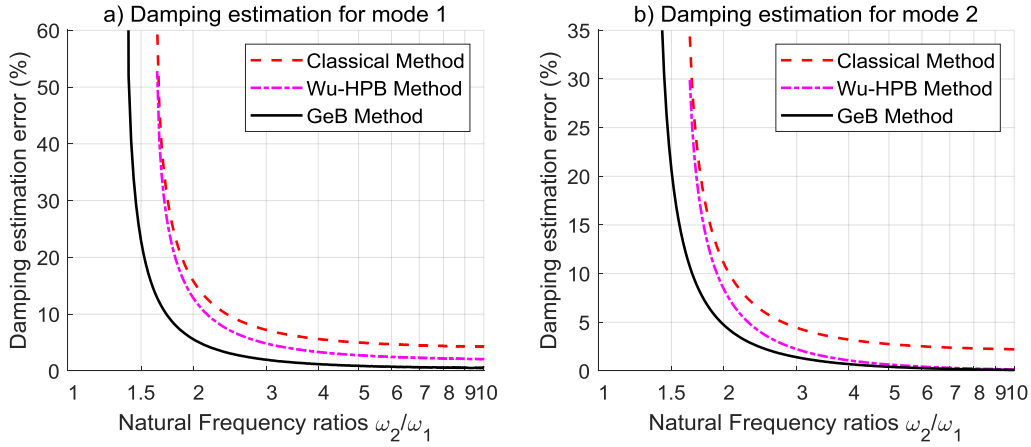

Fig. 5. Dependence of damping estimation error on the natural frequency ratio ω_2/ω_1 (with $\xi_1 = \xi_2 = 0.10$)

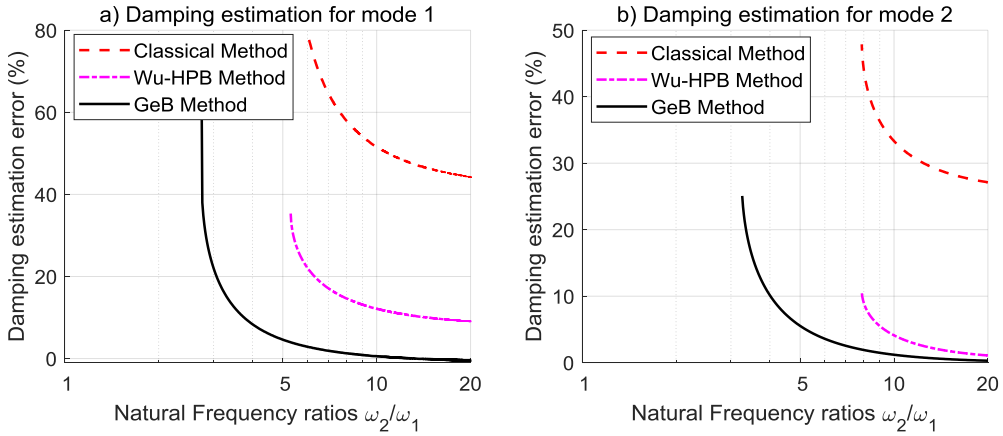

Fig. 6. Dependence of damping estimation error on the natural frequency ratio ω_2/ω_1 (with $\xi_1 = \xi_2 = 0.30$).

On the other hand, the estimation range by the GeB method is also larger than the formulas based on the HPB method. In Fig. 5, the GeB method can estimate the damping when the natural frequency ratio ω_2/ω_1 is about 1.2, while the HPB method can only do the same thing when the natural frequency ratio ω_2/ω_1 is greater than 1.6. Furthermore, for high damping levels (Fig. 6), the damping estimation ranges of the first and second modes by the GeB method are about $\omega_2/\omega_1 \geq 2.6$ and $\omega_2/\omega_1 \geq 3.1$, respectively. Meanwhile, the corresponding ranges are about $\omega_2/\omega_1 \geq 5$ and $\omega_2/\omega_1 \geq 8$ by HPB method using Wu's formula.

4.3. Effect of the amplitude ratio a_2/a_1 on the damping estimation error

For varying amplitude ratio a_2/a_1, investigate the error in estimating the damping ratios of the first and second modes using the above formulas (with natural frequency ratio $\omega_2/\omega_1 = 4$). The results are given in Fig. 7 and Fig. 8 with damping ratios $\xi_1 = \xi_2 = 0.10$ and $\xi_1 = \xi_2 = 0.30$, respectively.

The larger the amplitude coefficient of a certain mode, the smaller the damping identification error of that mode. In some cases, the error depends on the amplitude ratio and it can be positive or negative and has no clear rule (Fig. 7b and Fig. 8a). However, in most cases, the damping estimation error by the GeB method is often smaller than the damping estimation error by the formulas of the HPB method.

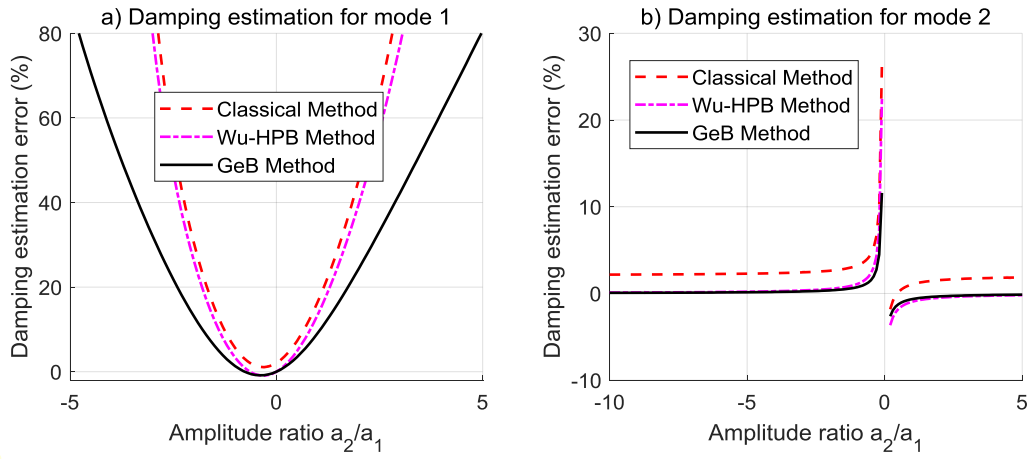

Fig. 7. Dependence of damping estimation error on the amplitude ratio a_0/a_1 (with $\xi_1 = \xi_2 = 0.10$).

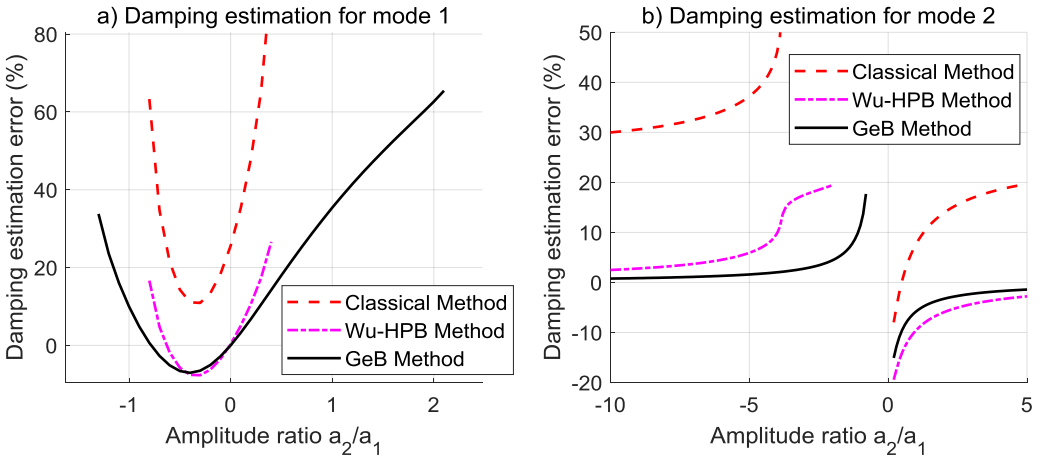

Fig. 8. Dependence of damping estimation error on the amplitude ratio a_2/a_1 (with $\xi_1 = \xi_2 = 0.30$).

5. Conclusion

The accuracy of the GeB and HPB methods in identifying viscous damping in the MDOF system from the frequency response function has been studied. The damping level, natural frequency ratio and amplitude coefficients have a great influence on the damping identification results of the methods. The larger the damping level, the higher the identification errors. The damping level of a certain mode greatly affects the damping estimation error of itself but does not affect other modes too much. The natural frequency ratio of the two modes has a similar influence on the damping estimation error of those two modes. When the natural frequency ratio decreases, the damping identification errors of both modes increase. The larger the amplitude coefficient of a certain mode, the smaller the damping identification error of that mode.

The analysis results also show that the GeB method has a smaller damping estimation error than the formulas of the HPB method. The GeB method also has the largest damping identification range, it can be applied in MDOF systems with low and high damping levels, and with close natural frequencies.

Acknowledgement

This research is funded by Program KC-4.0 under the grand number KC-4.0-22/19-25.

References

[1] A. K. Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, 4/E, Prentice Hall, Boston, 2011.

[2] D. J. Ewins, *Modal Testing: Theory, Practice, and Application*, 2nd Ed., Research Studies, New York, Chap. 3, 4, 2011.

[3] H. P. Yin, "A new theoretical basis for the bandwidth method and optimal power ratios for the damping estimation", *Mechanical Systems and Signal Processing*, Vol. 22, pp. 1869-1881, 2008. DOI: 10.1016/j.ymssp.2008.01.011

[4] H. P. Yin, "An average inverse power ratio method for the damping estimation from a frequency response function", *Mechanical Systems and Signal Processing*, Vol. 24, pp. 617-622, 2010. DOI: 10.1016/j.ymssp.2009.10.019

[5] B. A. Olmos and J. M. Roesset, "Evaluation of the half-power bandwidth method to estimate damping in systems without real modes", *Earthquake Engineering & Structural Dynamics*, Vol. 39, No. 14, pp. 1671-86, 2010. DOI: 10.1002/eqe.1010

[6] I. Wang, "An Analysis of Higher Order Effects in the Half Power Method for Calculating Damping", *Journal of Applied Mechanics*, Vol. 78, No. 1, 2011, 014501. DOI: 10.1115/1.4002208

[7] G. A. Papagiannopoulos and G. Hatzigeorgiou, "On the use of the half-power bandwidth method to estimate damping in building structures", *Soil Dynamics and Earthquake Engineering*, Vol. 31, pp. 1075-1079, 2011. DOI: 10.1016/j.soildyn.2011.02.007

[8] J. Wang, F. Jin, and C. Zhang, "Estimation error of the half-power bandwidth method in identifying damping for MDOF systems", *Soil Dynamics and Earthquake Engineering*, Vol. 39, pp. 138-142, 2012. DOI: 10.1016/j.soildyn.2012.02.008

[9] J. Wang, D. Lü, F. Jin, and C. Zhang, "Accuracy of the half-power bandwidth method with a third-order correction for estimating damping in MDOF systems", *Earthquake Engineering And Engineering Vibration*, Vol. 12, pp. 33-38, 2013. DOI: 10.1007/s11803-013-0149-1

[10] B. Wu, "A correction of the half-power bandwidth method for estimating damping", *Arch Appl Mech*, Vol. 85, pp. 315-320, 2015. DOI: 10.1007/s00419-014-0908-0

[11] Vũ Đình Hương, Lê Anh Tuấn, và Nguyễn Văn Trọng, "Phương pháp độ rộng dải tần số tổng quát trong bài toán nhận dạng cản nhớt của kết cấu", *Tạp chí Xây dựng*, Số 03/2015, tr. 61-63, 2015.

[12] Vũ Đình Hương và Phạm Đình Ba, "Nâng cao độ chính xác của phương pháp nhận dạng cản nội ma sát từ số liệu đo hàm phản ứng tần số gia tốc của kết cấu", *Tạp chí Xây dựng*, Số 04/2015, tr. 70-72, 2015.

[13] B. Wu, W. Liu, and X. Wu, "A damping estimation method based on power ratio", *Arch Appl Mech*, Vol. 88, pp. 1919-1927, 2018. DOI: 10.1007/s00419-018-1434-2

[14] Vũ Đình Hương, "*Nhận dạng cản trong dao động của kết cấu công trình*", Luận án Tiến sĩ kỹ thuật, Học viện KTQS, Hà Nội, 2016.

SỬ DỤNG PHƯƠNG PHÁP ĐỘ RỘNG DẢI TẦN SỐ TỔNG QUÁT NHẬN DẠNG CẢN TRONG HỆ HỮU HẠN BẬC TỰ DO

Vũ Đình Hương[1]

[1]*Viện Kỹ thuật công trình đặc biệt, Trường Đại học Kỹ thuật Lê Quý Đôn, Hà Nội, Việt Nam*

Tóm tắt: Phương pháp độ rộng dải tần số tổng quát là một trong những phương pháp để nhận dạng cản nhớt của kết cấu. Trong phương pháp này, một số công thức gần đúng và chính xác đã được nghiên cứu để ước lượng tỉ số cản trong hệ một bậc tự do. Tuy nhiên, khả năng áp dụng của các công thức này để ước lượng cản trong hệ hữu hạn bậc tự do vẫn chưa được phân tích một cách chi tiết. Bài báo nghiên cứu ảnh hưởng của các tham số dao động riêng tới độ chính xác của phương pháp độ rộng dải tần số tổng quát khi nhận dạng tỉ số cản nhớt trong hệ hữu hạn bậc tự do. Các kết quả được so sánh với các công thức đã có trên cơ sở phương pháp độ rộng nửa công suất cho thấy hiệu quả tốt và phạm vi ứng dụng rộng của phương pháp độ rộng dải tần số tổng quát.

Từ khóa: *Nhận dạng cản; phương pháp độ rộng dải tần số tổng quát; phương pháp độ rộng nửa công suất; hệ hữu hạn bậc tự do; hàm phản ứng tần số.*

Received: 02/04/2024; Revised: 18/06/2024; Accepted for publication: 28/06/2024

❑