STUDY ON THE VELOCITY POTENTIAL FIELD OF THE GROUP OF PARALLEL LONG CYLINDRICAL CHARGES

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Abstract

Previous theoretical studies have addressed the velocity potential of single charges. In practice, the group of explosive charges are commonly used. In this paper, the velocity potential field of a group of parallel long cylindrical charges is established by analytical method. The calculation program of the velocity potential of any element in the environment caused by a group of long cylindrical charges is written based on the Matlab programming language. This program allows to survey the velocity potential around a group of parallel long cylindrical charges. The analytical results showed that this velocity potential is on a plane and with the same specific conditions of the rockmass and the same blasting condition. Surveying results also showed that, the velocity potential of a group of parallel long cylindrical charges also decreases gradually as far as the case of single cylindrical charge. When the distance between the long cylindrical charges is reasonable, it will form an area, which has the velocity potential field close to the plane. This is the flat explosion wave area. This area has good implications for controlling the uniform breaking quality of rocks.

Keywords: Effectiveness of explosion; explosion; blasting design; explosive wave; parallel long cylindrical charges.

1. Introduction

According to the theory of hydrodynamics of explosion, when blasting is conducted, the energy of the explosion will move the environmental elements and wherever the velocity of this motion exceeds the velocity limit, there will be destruction there [1, 2, 3, 4, 5]. In order to determine the demolition zones and demolition properties, it is necessary to know the velocity distribution field of rock particles when exploding. This field again depends on the velocity potential field. Because the velocity in any direction is equal to the derivative of the velocity potential in that direction.

The principle of explosion in general, especially the hydrodynamic theory of explosion in rock and soil, has only mentioned the velocity field and the velocity potential field of a single charge, while in fact, the system of charges in parallel boreholes are widely applied [1, 2, 5]. This lack of reasoning limits the process of optimal control of the quality of rock fragmentation in practice. Therefore, the study on
establishing the velocity potential field of a group of parallel long cylindrical charges on
the same plane is an urgent scientific task.

2. Theoretical basis of the velocity potential

Analyzing the velocity potential of a single charge in rocky environment
according to the hydrodynamic theory of explosion

The boundary conditions of the problem are as follows:
- The environment of rock and soil is continuous, non-compressed and
  homogeneous;
- Explosion effect is carried out immediately.

According to the law of conservation of mass, the continuity equation of motion
shows the relationship of the value and direction of the velocity vector at any position
corresponding to the change of the environmental density, which is considered in the
three-dimensional space of the Cartesian coordinate system as follows:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0
\]  

(1)

where \( \rho \) - environmental density; \( u, v, w \) - the velocity of the environmental elements is
determined as follows:

\[
u = -\frac{\partial \phi}{\partial x}; \quad v = -\frac{\partial \phi}{\partial y}; \quad z = -\frac{\partial \phi}{\partial z}, \quad (m/s)
\]  

(2)

In many cases, it is possible to consider the continuous environment (in a solid
and liquid state) to be uncompresses when ignoring a small change in its volume, the
derivative of the density over time will be zero:

\[
\frac{\partial \rho}{\partial t} = 0
\]

Then equation (1) is shortened:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  

(3)

So the function \( \phi \) is called the velocity potential, O.E. Vlasov has set the velocity
potential of a single charge, which has an ellipsoid form with semi-axis lengths \( a, b, c \) and
in the case of long cylindrical charge (\( a = c \)) the function \( \phi \) has the value equal to [2]:

\[
\phi = \frac{2A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda}}, \quad (m^2/s)
\]  

(4)

where \( A \) is a constant determined by the formula:
Analysis of expression (4) finds that the velocity potential $\varphi$ at a point in the environment caused by a single long cylindrical charge depends on the energy of charge ($Q$), the size of charge ($a, b$), the position of a point to calculate the velocity potential ($\lambda$ is expressed by the coordinates $x, y, z$), the environmental density ($\rho$). In practice, the diameter of charge ($2a$) is usually fixed during the production of explosives to suit the diameter of the drill, and the length of charge may vary depending on the purpose of the job. Therefore, we can investigate the dependence of function $\varphi$ according to (4) on the length of charge ($b$) at a fixed position as Fig. 1 when the charge diameter of an explosive is given, under the same environmental conditions. From the graph in Fig. 1, we see that when $b$ increases, $\varphi$ also increases but only increases rapidly in the range $b = (5 \div 100)a$, then when $b$ continues to increase, the value of $\varphi$ gradually decreases. If the charge and the environment are constant, $\varphi$ depends on the position of a point to calculate the speed potential or the value of $\lambda$. When $\lambda = 0$, $\varphi$ reaches the maximum value at the surface of charge. On the other hand, we have:

$$\lim_{{\lambda \to \infty}} \ln \left( \frac{b^2 + \lambda + \sqrt{b^2 - a^2}}{a^2 + \lambda} \right) = 0$$

i.e. when $\lambda \to \infty$, or $x, y, z \to \infty$ then $\varphi = 0$. Thus, the further the distance from the charge is, the lower the $\varphi$ decreases and at a distance far enough then $\varphi = 0$.

Fig. 1. The velocity potential of long cylindrical charge is caused at one point when the diameter of charge is fixed and changing the length of charge.
Establishing the velocity potential field caused by a group of parallel long cylindrical charges in the same plane.

Consider a group of long cylindrical charges \( A_i, B_j \) \((i = 0\div m; j = 0\div n)\) in the form of an ellipsoid, with semi-axis lengths \( a, b, c \) \((a = c, b >> a)\) placed parallel and equidistant from each other at a distance \( l \) in an environment with an environmental density \( \rho \), where \( O \) is the origin of the Cartesian coordinates \( Oxyz \). Consider any point \( M \) in an environment with coordinates \((x, y, z)\) and its projection on two planes as shown in Fig. 2.

![Fig. 2. Model of a group of long cylindrical charges \( A_i, B_j \) and projection of point \( M \)](image)

Call by \( M' \) and \( M'' \) are the projection of the point \( M \) on the planes \( Oxz \) and \( Oxy \), respectively; \( \beta_j \) and \( \gamma_j \) \((j = 0\div n)\) are angled by the vector \( B_jM \) with the planes \( Oxz \) and \( Oxy \), respectively; \( \alpha_{Bj} \) and \( \theta_{Bj} \) \((j = 0\div n)\) are respectively angled by the vectors \( B_jM' \) and \( B_jM'' \) with the axes Ox and Oy, respectively. We have:

\[
\cos \beta_j = \frac{B_jM'}{B_jM} = \frac{\sqrt{(x-jL)^2 + z^2}}{\sqrt{(x-jL)^2 + y^2 + z^2}};
\]

\[
\cos \gamma_j = \frac{B_jM''}{B_jM} = \frac{\sqrt{(x-jL)^2 + y^2}}{\sqrt{(x-jL)^2 + y^2 + z^2}}
\] (5)
On the other hand:

\[
\cos \alpha_{ Bj} = \frac{x - j l}{\sqrt{(x - j l)^2 + z^2}}; \quad \sin \alpha_{ Bj} = \frac{z}{\sqrt{(x - j l)^2 + z^2}}; \quad \cos \theta_{ Bj} = \frac{y}{\sqrt{(x - j l)^2 + y^2}} \quad (6)
\]

We have:

\[
B_{ j} M_{ x} = B_{ j} M_{ }. \cos \alpha_{ Bj} = B_{ j} M_{ }. \cos \beta_{ j}. \cos \alpha_{ Bj}
\]

\[
= B_{ j} M_{}. \sqrt{(x - j l)^2 + z^2}, \quad \frac{x - j l}{\sqrt{(x - j l)^2 + y^2 + z^2}}, \quad \frac{x - j l}{\sqrt{(x - j l)^2 + y^2 + z^2}}
\]

The velocity potential due to the charge \(B_{ j}\) caused at \(M\) by formula (4) is valid:

\[
\varphi_{ Bj,M} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ Bj}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ Bj}}} \quad (8)
\]

Substituting (8) into formula (7) and doing the same with the axes \(y\) and \(z\), we should have:

\[
B_{ j} M_{ x} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ Bj}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ Bj}}} \cdot \frac{x - j l}{\sqrt{(x - j l)^2 + y^2 + z^2}}
\]

\[
B_{ j} M_{ y} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ Bj}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ Bj}}} \cdot \frac{y}{\sqrt{(x - j l)^2 + y^2 + z^2}}
\]

\[
B_{ j} M_{ z} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ Bj}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ Bj}}} \cdot \frac{z}{\sqrt{(x - j l)^2 + y^2 + z^2}}
\]

The charges to the left of point \(O\) are calculated similarly:

\[
A_{ M_{ x}} = A_{ M}. \frac{x + i l}{\sqrt{(x + i l)^2 + y^2 + z^2}} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ A_{ l}}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ A_{ l}}}} \cdot \frac{x + i l}{\sqrt{(x + i l)^2 + y^2 + z^2}}
\]

\[
A_{ M_{ y}} = A_{ M}. \frac{y}{\sqrt{(x + i l)^2 + y^2 + z^2}} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ A_{ l}}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ A_{ l}}}} \cdot \frac{y}{\sqrt{(x + i l)^2 + y^2 + z^2}}
\]

\[
A_{ M_{ z}} = A_{ M}. \frac{z}{\sqrt{(x + i l)^2 + y^2 + z^2}} = \frac{2 A}{\sqrt{b^2 - a^2}} \ln \frac{\sqrt{b^2 + \lambda_{ A_{ l}}} + \sqrt{b^2 - a^2}}{\sqrt{a^2 + \lambda_{ A_{ l}}}} \cdot \frac{z}{\sqrt{(x + i l)^2 + y^2 + z^2}}
\]

The combined velocity potential caused by all the charges \(A_{ i}\) and \(B_{ j}\) at \(M\) is calculated as follows:

\[
\bar{\varphi}_{ M} = \sum_{i=1}^{n} \bar{\varphi}_{ A_{ i}} + \sum_{j=0}^{n} \bar{\varphi}_{ B_{ j}}
\]

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or:
\[ \phi_M = \left( \sum_{i=1}^{m} A_M x_i + \sum_{j=0}^{n} B_M z_j \right) + \left( \sum_{i=1}^{m} A_M y_i + \sum_{j=0}^{n} B_M z_j \right) + \left( \sum_{i=1}^{m} A_M z_i + \sum_{j=0}^{n} B_M y_i \right) \] \hspace{1cm} (12)

Since the points \( M \) are all on the ellipsoid, with the center of the ellipsoid being the center of the charge, the values \( \lambda_{Al} \) and \( \lambda_{Bj} \) are found from the following condition:
\[ \frac{(x - j l)^2}{a^2 + \lambda_{Bj}} + \frac{z^2}{b^2 + \lambda_{Bj}} = 1; \quad j = 0 + n; \]
\[ \frac{(x + i l)^2}{a^2 + \lambda_{Al}} + \frac{z^2}{b^2 + \lambda_{Al}} = 1; \quad i = 1 + m \] \hspace{1cm} (13)

Solving equation (13) with unknowns \( \lambda_{Al} \) and \( \lambda_{Bj} \) we get:

\[ \lambda_{Bj} = \frac{x^2}{2} + \frac{y^2}{2} + \frac{a^2}{2} - \frac{b^2}{2} + \frac{j^2 l^2}{2} - j l x + (a^4 - 2 a^2 b^2 - 2 a^2 j^2 l^2 + 4 a^2 j l x - 2 a^2 x^2 + 2 a^2 y^2 - 2 a^2 x^2 + 2 b^2 y^2 + 2 b^2 z^2 + j^4 l^4 + 4 j^4 l^2 x + 6 j^2 l^2 x^2 + 2 j^2 l^2 y^2 + 2 j^2 l^2 z^2 - 4 j l x^3 - 4 j l y z^2 - 4 j l x z^2 + x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4) / 2; \]

\[ \lambda_{Al} = \frac{x^2}{2} + \frac{y^2}{2} + \frac{a^2}{2} - \frac{b^2}{2} - \frac{j^2 l^2}{2} + j l x + (a^4 - 2 a^2 b^2 - 2 a^2 j^2 l^2 + 4 a^2 j l x - 2 a^2 x^2 + 2 a^2 y^2 - 2 a^2 x^2 + 2 b^2 y^2 + 2 b^2 z^2 + j^4 l^4 + 4 j^4 l^2 x + 6 j^2 l^2 x^2 + 2 j^2 l^2 y^2 + 2 j^2 l^2 z^2 + 4 j l x^3 + 4 j l y z^2 + 4 j l x z^2 + x^4 + 2 x^2 y^2 + 2 x^2 z^2 + y^4 + 2 y^2 z^2 + z^4)^{(1/2)} / 2 \] \hspace{1cm} (14)

Thus, the value of the velocity potential at point \( M \) is:
\[ \varphi_M = \left( \sum_{i=1}^{m} A_M x_i + \sum_{j=0}^{n} B_M z_j \right)^2 + \left( \sum_{i=1}^{m} A_M y_i + \sum_{j=0}^{n} B_M z_j \right)^2 + \left( \sum_{i=1}^{m} A_M z_i + \sum_{j=0}^{n} B_M y_i \right)^2 ; \]

\[ i = 1 + m; \quad j = 0 + n \] \hspace{1cm} (15)

In which the values \( A_M x, A_M y, A_M z, B_M x, B_M y, B_M z \) are determined from the equations (9), (10), (14).

Thus, the expression (15) is the equation describing the velocity potential of the group of long cylindrical charges to be placed parallel to each other in the same plane.

Equation (15) is a general equation for determining the velocity potential of a point in the environment of the group of parallel long cylindrical charges in the same plane. The velocity potential caused by a group of charges depends on the parameters of a single charge \( (Q, \lambda, \rho, a, b) \) and depends on the distance between the charges to each other \( (l) \) and the number of charges \((i, j)\). Determining the velocity potential field by direct calculation is complex. Therefore, in order to be convenient in researching,
applications calculation, investigating the velocity potential field of the group of long cylindrical charges, it is necessary to develop a calculation program.

3. Establishing the program to calculate the velocity potential, survey and analysis

3.1. Program description

Using the Matlab programming language in building a computer program to calculate the velocity potential field called "Thetocado". The input data to calculate include: parameters of charge, the number of charges, the distance between the charges, the environmental density. From these data, the computer will output the velocity potential field value at the location that we choose to study.

When determining the velocity potential at any point caused by the group of charges, from formula (2), we can determine the velocity of the environmental element at that position. Based on the data on the critical velocity that caused the environment to be destroyed, we can check whether the location is destroyed or not. In addition, based on the determined velocity potential field, based on the theory of O.E. Vlasov we can determine the relative size of the area that is likely to be destroyed.

3.2. Surveying by numerical method and analysis

Use the established program to calculate the velocity potential field of the group of long cylindrical charges, with the following data:

- The long cylindrical charge of TNT can be considered as a ellipsoid with semi-axis lengths \( a = c = 0.03 \) m; \( b = 1.5 \) m. Average density of explosive: 1580 kg/m\(^3\). The energy when releasing one kilogram of TNT is 1000 kcal.
- Number of charges to be surveyed: 20 charges.
- Environmental density: 200 kg.s\(^2\)/m\(^4\).
- Surveying with three cases:
  + The distance between charges is: 0.1 m.
  + The distance between charges is: 1.5 m.
  + The distance between charges is: 15 m.
- Conducting the survey at two positions including \( y = 0 \) m (through the center of charge) and \( y = 1.5 \) m (at the beginning of charge).

Survey results are shown in Fig. 3, Fig. 4, Fig. 5.
Fig. 3. The velocity potential field of the group of 20 long cylindrical charges, 
the charges equidistant from each other at a distance of 0.1 m
(sign F (m²/s) is the velocity potential value of equipotential line)
a) At the cross section through the center of charge (y = 0 m);
b) At the cross section through the beginning of charge (y = 1,5 m)

Fig. 4. The velocity potential field of the group of 20 long cylindrical charges, 
the charges equidistant from each other at a distance of 1,5 m
(sign F (m²/s) is the velocity potential value of equipotential line)
a) At the cross section through the center of charge (y = 0 m);
b) At the cross section through the beginning of charge (y = 1,5 m)
Fig. 5. The speed potential field of the group of 20 long cylindrical charges, the charges velocity equidistant from each other at a distance of 15 m (sign $F$ (m$^2$/s) is the speed potential value of equipotential line)

a) At the cross section through the center of charge ($y = 0$ m);
b) At the cross section through the beginning of charge ($y = 1.5$ m)

Thus, the velocity potential caused by a group of parallel long cylindrical charges depends on the parameters of a long cylindrical charge (as commented in section 2.1), on the distance between the charges and the number of charges. When the parallel long cylindrical charges on the same plane (explosion plane), the velocity potential field caused by this group of charges has the following characteristics:

- If the distance between the charges is small:
  + When the number of charges is not many or the size of the explosion plane is small, the equipotential lines in the velocity potential field are similar to the cases of concentrated charge.

  + When the number of charges is small, the size of the explosion plane is many times larger than the diameter of the charge, the equipotential lines in the velocity potential field will be similar to the case of the flattened charge (slab charge).

- If the distance between the charges is large:
  + When the distance between the charges increases, the velocity potential will decrease, the region of equipotential lines parallel to the explosion plane will be formed at an appropriate distance.

  + When the distance between the charges is enlarged to a certain extent, only the equipotential lines around the charge appear, the value of the equipotential lines of the explosion plane is very small or in other words, the influence of the charges is negligible.
4. Conclusion and recommendation

The establishment of the velocity potential field of the group of long cylindrical charges is based for determine the mechanical characteristics of the explosion efficiency and the interaction between the charges more clearly, by determining the combined velocity potential caused by single charges.

The velocity potential of a long cylindrical charge caused at a position depends on the parameters of the charge (diameter, length of charge, type of explosive), the position of the survey point and environmental characteristics.

The velocity potential caused by a group of long cylindrical charge depends not only on the parameters of the charge and the environment, as in the case of a single long cylindrical charge, but also on the distance between the charges and the number of charges.

When the distance between the long cylindrical charges is closer together, the velocity potential field of the group of long cylindrical charges will be the more valuable. When the distance between the charges is large to a certain extent, the influence between the charges is negligible (Fig. 5), equipotential lines are formed mainly by the single charges.

The velocity potential field reaches its maximum value at the cross section which cuts through the center of charge and it is perpendicular to the axis of the long cylindrical charge and decreases gradually with distance from the center of charge. This shows that the length of charge also affects the velocity potential field.

When charges are arranged at close distances to each other, near the plane which sets the long cylindrical charges will appear equipotential lines almost parallel to each other and parallel to that plane. This is important in practice because when there are parallel equipotential lines, a flat blast wave has been formed, which is characterized by the decrease more slowly than the cylindrical blast wave and the spherical explosive wave, it makes the rock smashed more evenly. On the other hand, when parallel equipotential lines appear, the environmental elements move in the same direction with each other in the normal direction with the plane containing long cylindrical charges, so it is very convenient for application in controlling flying direction of rock when blasting.

The results of this research are recommended to propose solutions in controlling the explosion parameters to ensure the appearance of a reasonably flat explosive wave zone, to improve the quality of rock fragmentation and control the flying direction of rock.
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NGHIÊN CỨU THIẾT LẬP TRƯỞNG THẾ TỔC ĐỘ CỦA NHÓM LƯỢNG NỞ DÀI SONG SONG


Từ khóa: Hiệu quả nở; nở; thiết kế nở min; sóng nở; lượng nở dài song song.

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