GENERALIZED GROUP DETECTION FOR MASSIVE MIMO SYSTEMS

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Abstract
In this paper, we propose a Generalized Group Detection (GGDex) algorithm for signal recovery in Massive MIMO systems. The algorithm is built by dividing the extended Massive MIMO system into several sub-systems, which then can be detected by any classical detectors in a successive-interference-cancellation manner. Based on the GGDex, we further propose 4 efficient detectors called the ZF-GGDex and SQRD-GGDex, the ZF-Presorted GGDex and the SQRD-Presorted GGDex ones. The analytical and simulation results show that all proposed detectors can achieve more than 3dB in BER performance compared to those of the conventional linear ones while their complexities are kept at reasonable levels. Particularly, performances of the ZF-Presorted GGDex and the SQRD-Presorted GGDex approach tightly those of the BLAST. Consequently, they are very good candidates to use in Massive MIMO systems.

Keywords: Massive MIMO; Up-link; Low complexity receiver; Generalized Group Detection.

1. Introduction
In recent years, Massive Multi-Input-Multi-Output (Massive MIMO) systems have been proposed to improve data rate and reliability of wireless communication systems. In Massive MIMO systems, hundreds of antennas are placed at the Base Station (BS) to serve simultaneously tens of users using the same frequency resource. By utilizing a large number of antennas at the BS, Massive MIMO systems can provide a huge throughput and high energy/spectral efficiencies [1], [2].

In Massive MIMO systems, low complexity detectors, such as the Zero-Forcing (ZF) and the Minimum Mean Square Error (MMSE) or the detectors based on QR decomposition algorithm (i.e. the QRD and the Sorted QRD detectors) are preferred due to their low complexities. In fact, as the load factor \( \beta \) (defined by the ratios of total transmit antennas and receive ones. This means \( \beta = \frac{KN_r}{N_r} \), where \( K \), \( N_r \) and \( N_r \) respectively denotes the number of active users, the number of antennas placed at each user and the ones equipped at the BS) is sufficiently small, Bit Error Rate (BER) performances of the linear detectors are able to reach that of the Maximum Likelihood
(ML) detector [1]. However, when $\beta$ approaches unity, the performances of these detectors are reduced significantly. Therefore, for the systems with high $\beta$ (i.e. $\beta \approx 1$), it is of necessity to develop new detectors, whose performances are higher than those of the ZF, the MMSE or the QRD/SQRD detectors at reasonable complexity levels.

A method of reducing the detection complexity is to use the Group Detection (GD) algorithm [3], which is first proposed to reduce the complexity of the ML detector adopted in the conventional MIMO systems. In [4] the authors proposed the iterative group detection and decoding scheme for large coded-MIMO systems. By combining the soft-output detection and the group detection technique, this proposed scheme can improve BER performance of coded-MIMO systems significantly. However, the proposed detection scheme is for point-to-point MIMO systems. Besides, only small scale fading is investigated. Therefore, the technique cannot be directly applied to Massive MIMO systems, where wireless channel is strongly affected by large scale fading. Recently, Nguyen et al. proposed the so-called Zero-Forcing detector based Group Detection (ZF-GD) [2], the ZF detector based on the GD algorithm for Massive MIMO, which has not only higher BER performance, but also lower complexity as compared to the classical ZF. However, it still underperforms the classical MMSE one when $\beta$ approaches unity.

In this paper, we apply the Group Detection concept to the equivalent extended form of the Massive MIMO channel matrix and generalize it so that an arbitrary number of sub-systems can be generated. The proposed approach is referred to as the Generalized Group Detection Algorithm applied to the extended system or GGDex for short. In the sub-systems generated by the GGDex, either the classical ZF or the SQRD can be adopted to recover the transmitted signals, resulting in the two proposed detectors called the ZF-GGDex and the SQD-GGDex. In order to further improve the BER performances of the ZF-GGDex and the SQD-GGDex detectors, we propose a sort procedure applied to the extended channel matrix before the GGDex is carried out. As a consequence, two new detectors correspondingly called the ZF-Presorted GGDex and the SQRD-Presorted GGDex, can be obtained. The simulation results show that at high load factors the proposed detectors noticeably outperform their MMSE counterpart while their computational complexities are kept at comparable levels. Particularly, performances of the ZF-Presorted GGDex and the SQRD-Presorted GGDex approach tightly those of the BLAST one. Consequently, they are very good candidates for signal detection in Massive MIMO systems.
The rest of the paper is organized as follows: Section 1 illustrates the Massive MIMO system model. In Section 2, we present the Generalized Group Detection algorithm for Massive MIMO systems as well as the proposed ZF-GGDex, SQRD-GGDex, ZF-Presorted GGDex and SQRD-Presorted GGDex detectors. The complexity analysis and performance comparison are presented in Section 3. Finally, Section 4 concludes the paper.

Notations: \( \mathbb{C} \) denote the sets of complex numbers; \( Q \) is quantization operation, which slices the estimated symbols to the nearest integer values associated with the transmitted QAM constellation. In this paper, Matlab notations are used to define vectors and matrices. For example: \( B = A(:, i : j) \) indicates that matrix \( B \) is generated by taking from column \( i \) to column \( j \) of \( A \); \( I_N \) is \( N \times N \) identity matrix, and \( 0_{N \times 1} \) is the \( N \times 1 \) zero vector; \( (\cdot)^T \) and \( (\cdot)^H \) denote transpose and Hermitian transpose operations, respectively; \( E[\cdot] \) denotes expectation operation; \( A^+ = (A^H A)^{-1} A^H \) is the pseudo-inverse of matrix \( A \) and \( \otimes \) is Kronecker product.

2. System model

Let us consider a Massive MIMO system as depicted in Fig. 1. The system includes a BS equipped with \( N_T \) antennas to serve \( K \) active users, each of them equipped with \( N_R \) antennas. The cell is assumed to be circular, where the BS is located at the cell’s origin. All active users are distributed randomly in the cell such that the distance \( d_k \) from the \( k \)th user \( (k = 1, 2, ..., K) \) to the BS satisfies \( d_0 \leq d_k \leq r \). Herein, \( d_0 \) and \( r \) respectively denote the reference distance and the cell’s radius.
The received signal vector at the BS is given as follows:

\[
\mathbf{y} = \sqrt{\frac{p^{(ul)}}{N_f E_s}} \mathbf{H} (\mathbf{A} \otimes \mathbf{N}_T)^{1/2} \mathbf{x} + \mathbf{n}
\]  

(1)

where \( E_s \) is the average symbol energy of the M-QAM signals; \( p^{(ul)} \) denotes the average transmit power of each user; \( \mathbf{H} \in \mathbb{C}^{N_r \times N} \), \( N = KN_T \), is the channel matrix, whose entries are small scale fading coefficients; and \( \mathbf{n} \in \mathbb{C}^{N \times 1} \) denotes the noise vector. The entries of \( \mathbf{H} \) and \( \mathbf{n} \) are assumed to be independent identically distributed random variables with zero mean and unit variance; \( \mathbf{x} \in \mathbb{C}^{N \times 1} \) is the transmit signal vector composed of all active users’ signals satisfying \( E[\mathbf{x}\mathbf{x}^H] = E_s \mathbf{I}_N \); \( \mathbf{A} \in \mathbb{C}^{N \times N} \) is a diagonal matrix, whose diagonal entries, \( a_k, k = 1, 2, \ldots, K \), represents the large scale fading coefficients between \( k \)th user and the BS. Herein, \( a_k \) is determined by [5]

\[
a_k = \frac{z_k}{d_k} \gamma
\]  

(2)

where \( \gamma \) is the path loss component, \( z_k \) is a random variable with zero mean and variance \( \sigma_{\text{Shadow}} \) representing the shadowing effect. Let us define

\[
\mathbf{U} = \left( \sqrt{\frac{p^{(ul)}}{N_f E_s}} \right) \mathbf{H} (\mathbf{A} \otimes \mathbf{N}_T)^{1/2} .
\]

Then, equation (1) can be rewritten as

\[
\mathbf{y} = \mathbf{Ux} + \mathbf{n}
\]  

(3)

The system equation in (3) can be equivalently rewritten in the extended form as

\[
\mathbf{y}_{ex} = \mathbf{U}_{ex} \mathbf{x} + \mathbf{n}_{ex}
\]  

(4)

where \( \mathbf{y}_{ex}, \mathbf{U}_{ex} \) and \( \mathbf{n}_{ex} \) are respectively the \( (N_r + N) \times 1 \) extended received vector, \( (N_r + N) \times N \) extended channel matrix and \( (N_r + N) \times 1 \) extended noise vector. They are defined by

\[
\mathbf{y}_{ex} = \begin{bmatrix} \mathbf{y}^T & 0_{N \times 1}^T \end{bmatrix}^T ; \quad \mathbf{U}_{ex} = \begin{bmatrix} \mathbf{U}^T & \sqrt{E_s} \mathbf{I}_N \end{bmatrix}^T \quad \text{and} \quad \mathbf{n}_{ex} = \begin{bmatrix} \mathbf{n}^T & -\sqrt{E_s} \mathbf{x}^T \end{bmatrix}^T .
\]

3. Detectors based on Generalized Group Detection algorithm

A. Generalized Group Detection algorithm

In this sub-section, we utilize Group Detection concept presented in [2] and generalize it for signal detection in Massive MIMO systems. In the proposed method, the transmitted signal vector is recovered in a stage-by-stage fashion, which is similar to the Successive Interference Cancellation (SIC) technique. However, the difference is
that at each stage, a new sub-system is generated allowing the transmitted symbols from one or more users to be detected simultaneously. After that, the interferences of these recovered symbols are canceled out from the received signal vector to generate a new received signal vector for the next stage. After the transmitted signals from all the stages are estimated, they are rearranged to give the final estimated vector. Let $L (2 \leq L \leq K)$ be the number of stages and $m_k$ be the number of users belonging to the $k$th sub-system. Without loss of generality, we assume that the users are divided equally among the sub-systems, i.e., $m_k = m_j = m, \forall j, k$ and $K = mL$. Let us define:

$$G_k = U_{ex}(:,(k-1)l+1:kl), G^{(k)} = U_{ex}(:,kl+1:N), s_k = x((k-1)l+1:kl), \text{ and } s^{(k)} = x(kl+1:N)$$

where $k = 1, 2...L$ and $l = mN_f$. The GGDex procedure is illustrated in Fig. 3 and described in details below.

First, we set $y^{(1)} = y_{ex}$ and rewrite (4) as follows:

$$y^{(1)} = y_{ex} = G_1s_1 + \sum_{k=2}^{L} G_ks_k + n_{ex} = Gs_1 + G^{(1)}s^{(1)} + n_{ex}. \quad (5)$$

Next, the first sub-system is generated by applying the orthogonal projection term of $G^{(1)}$, i.e., $P^{(1)} = I - G^{(1)}G^{(1)\dagger}$, to equation (1) as

$$P^{(1)}y^{(1)} = P^{(1)}G_1s_1 + P^{(1)}n_{ex} \quad (6)$$

or equivalently, we can write

$$\tilde{y}^{(1)} = \tilde{G}^{(1)}s_1 + \tilde{n}_{ex} \quad (7)$$

where $\tilde{y}^{(1)} = P^{(1)}y^{(1)}$, $\tilde{G}^{(1)} = P^{(1)}G_1$ and $\tilde{n}_{ex} = P^{(1)}n_{ex}$. The sub-vector $s_1$ can be estimated by applying any conventional detector to the sub-system in (7). As soon as the detection process completes, the recovered signal vector $\hat{s}_1$, which is assumed to be correctly detected, is used to cancel the interference effect of $s_1$ out of the received signal vector $y^{(1)}$ as follows:

$$y^{(2)} = y^{(1)} - G_1\hat{s}_1 = G^{(1)}s^{(1)} + n_{ex} = \sum_{k=2}^{L} G_ks_k + n_{ex} \quad (8)$$

The newly generated received signal vector $y^{(2)}$ in (8) can be re-expressed as:

$$y^{(2)} = G_2s_2 + \sum_{k=3}^{L} G_ks_k + n_{ex} = G_2s_2 + G^{(2)}s^{(2)} + n_{ex} \quad (9)$$
Again, using the projection term, \( \mathbf{P}^{(2)} = (\mathbf{I} - \mathbf{G}^{(2)} \mathbf{G}^{(2)*}) \) the 2nd sub-system is generated as follows:

\[
\hat{\mathbf{y}}^{(2)} = \hat{\mathbf{G}}^{(2)} \mathbf{s}_2 + \hat{\mathbf{n}}_{\text{ex}}^{(2)}
\]

where \( \hat{\mathbf{y}}^{(2)} = \mathbf{P}^{(2)} \mathbf{y}^{(2)} \), \( \hat{\mathbf{G}}^{(2)} = \mathbf{P}^{(2)} \mathbf{G}_2 \), and \( \hat{\mathbf{n}}_{\text{ex}}^{(2)} = \mathbf{P}^{(2)} \mathbf{n}_{\text{ex}} \). The above processes are repeated until the transmitted signals of the \( L - 1 \) stages are recovered. In general, the \( k \)th sub-system equation, \( 1 \leq k \leq (L - 1) \), is given by:

\[
\hat{\mathbf{y}}^{(k)} = \hat{\mathbf{G}}^{(k)} \mathbf{s}_k + \hat{\mathbf{n}}_{\text{ex}}^{(k)}
\]

where \( \hat{\mathbf{y}}^{(k)} = \mathbf{P}^{(k)} \mathbf{y}^{(k)} \); \( \hat{\mathbf{G}}^{(k)} = \mathbf{P}^{(k)} \mathbf{G}_k \), \( \hat{\mathbf{n}}_{\text{ex}}^{(k)} = \mathbf{P}^{(k)} \mathbf{n}_{\text{ex}} \); \( \mathbf{P}^{(k)} = (\mathbf{I} - \mathbf{G}^{(k)} \mathbf{G}^{(k)*}) \). The received vector \( \mathbf{y}^{(k)} \) is defined as:

\[
\mathbf{y}^{(k)} = \begin{cases} 
\mathbf{y}_{\text{ex}}^{(k)} & \text{for } k = 1 \\
\mathbf{y}^{(k-1)} - \mathbf{G}_{k-1} \hat{\mathbf{s}}_{k-1} & \text{for } 1 < k \leq (L - 1)
\end{cases}
\]

It is worth noting that the last sub-system is generated just by removing the interference term \( \mathbf{G}_{L-1} \mathbf{s}_{L-1} \) out of \( \mathbf{y}^{(L-1)} \) to get:

\[
\mathbf{y}^{(L)} = \mathbf{y}^{(L-1)} - \mathbf{G}_{L-1} \hat{\mathbf{s}}_{L-1} = \mathbf{G}_L \hat{\mathbf{s}}_L + \mathbf{n}_{\text{ex}}
\]

Therefore, the transmitted signals from the remaining users in the last sub-system are detected simply by applying conventional detector to (13). Finally, the overall estimated vector \( \hat{\mathbf{x}} \) are obtained by rearranging all the estimated sub-vectors as 

\[
\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{s}}_1^T & \hat{\mathbf{s}}_2^T & \cdots & \hat{\mathbf{s}}_L^T \end{bmatrix}^T
\]

**B. ZF-GGDex detector**

The conventional ZF detector can be directly applied to either the original system in (3) or to the extended system in (4) to recover the transmitted signals at the BS. Note that if the ZF detector is adopted in the extended system, it is equivalent to MMSE detector. Hence, by applying the ZF detection procedure respectively to the systems in (3) and (4), we get:

\[
\hat{\mathbf{x}}_{\text{ZF}} = \mathbf{U}^* \mathbf{y} = \mathbf{x} + \mathbf{U}^* \mathbf{n}
\]

\[
\hat{\mathbf{x}}_{\text{MMSE}} = \mathbf{U}_{\text{ex}}^* \mathbf{y}_{\text{ex}} = \mathbf{x} + \mathbf{U}_{\text{ex}}^* \mathbf{n}_{\text{ex}}
\]

These signal vectors are then sliced to obtain the recovered signals as 

\[
\hat{\mathbf{x}}_{\text{ZF}} = \mathcal{Q}(\hat{\mathbf{x}}_{\text{ZF}}) \quad \text{and} \quad \hat{\mathbf{x}}_{\text{MMSE}} = \mathcal{Q}(\hat{\mathbf{x}}_{\text{MMSE}}).
\]

Linear detectors offer low complexities. Therefore, they are suitable for signal recovery in Massive MIMO systems. However, it is shown in [6] that linear detectors can only provide a diversity order \( (N_r - N + 1) \). As the load factor equals unity (i.e.,
the achievable diversity order is 1, implying that no diversity is achieved. In such a scenario, the performance of the ZF and MMSE detectors become very low, thereby making them less attractive.

By applying the conventional ZF detection procedure to (11) and (13), the proposed ZF-GGDex detector is realized. It estimates the transmitted sub-vector, $\hat{s}_k$, as

$$
\hat{s}_k = \begin{cases} 
Q(\tilde{\mathbf{G}}_k^{(k)}(\tilde{\mathbf{y}}_k^{(L)})) = Q(\mathbf{s}_k + \tilde{\mathbf{G}}_k^{(k)}\tilde{n}_k^{(L)}), & k < L \\
Q(\mathbf{G}_k^L\mathbf{y}_k^{(L)}) = Q(\mathbf{s}_k + \mathbf{G}_k^L\mathbf{n}_k), & k = L 
\end{cases}
$$

(16)

It is worth emphasizing that when the ZF-GGDex detector is used, the equivalent load factor given by each sub-system is given by $\beta_s = l/(N_r + N)$, which is much smaller than that of the original system (i.e., $\beta_s \ll \beta = (N/N_r)$). Therefore, the ZF-GGDex detector is capable of remarkably outperforming its conventional counterpart despite the fact that it suffers from the error propagation phenomenon, as demonstrated by the simulation results in the below section.

C. SQRD-GGDex detector

The proposed SQRD-GGDex detector is realized by applying the classical SQRD detector to (11) and (13). The transmitted signals of $k$th sub-system $\tilde{\mathbf{y}}^{(k)} = \tilde{\mathbf{G}}^{(k)}\mathbf{s}_k + \tilde{n}^{(k)}_k$, are recovered step by step as follows.

First, the channel matrix of the $k$th sub-system (i.e., $\tilde{\mathbf{G}}^{(k)}$, $k=1,2,...,L$) is decomposed by using the Sorted QR decomposition presented in [7] to provide the unitary matrix $\mathbf{Q}^{(k)} \in \mathbb{C}^{(N_r+N)^d}$, the upper triangle matrix $\mathbf{R}^{(k)} \in \mathbb{C}^{(d)}$ and the permutation vector $\mathbf{p}^{(k)}$.

Next, multiplying both sides of the $k$th sub-system $\tilde{\mathbf{y}}^{(k)}$ by $\mathbf{Q}^{(k)H}$ we obtain:

$$
\mathbf{v}^{(k)} = \mathbf{Q}^{(k)H}\tilde{\mathbf{y}}^{(k)} = \begin{bmatrix} 
\mathbf{R}^{(k)}\mathbf{s}_k + \mathbf{Q}^{(k)H}\tilde{n}^{(k)}_k, & k < L \\
\mathbf{R}^{(k)}\mathbf{s}_k + \mathbf{Q}^{(k)H}\tilde{n}^{(k)}_k, & k = L 
\end{bmatrix}
$$

(17)

Note here that $\tilde{\mathbf{G}}^{(L)} = \mathbf{G}_L$ and $\tilde{\mathbf{y}}^{(L)} = \mathbf{y}^{(L)}$. After that, all the $l$ transmitted symbols of the sub-vector $\mathbf{s}_k$ (denoted by $\hat{s}_{i,k}, i=1,2,...,l$) can be estimated by the following rule:

$$
\hat{s}_{i,k} = \begin{cases} 
Q\left( \frac{\mathbf{v}^{(k)}_{i}}{r_{i,k}} \right), & i = l \\
Q\left( \mathbf{v}^{(k)}_{i} - \sum_{j=i+1}^{l} \frac{r^{(k)}_{i,j} \hat{s}_{j,k}}{r_{i,j}} \right), & i = 1, \ldots, l-1 
\end{cases}
$$

(18)
where $v_{i}^{(k)}$ is the $i$th component of $v^{(k)}$; $r_{i,j}^{(k)}$ is the entry at the $i$th row, $j$th column of $R^{(k)}$. Finally, the estimated vector $\hat{s}_i$ is reordered according to $p^{(k)}$ as:

$$\hat{s}_i = \hat{s}_i(p^{(k)})$$

(19)

herein, $\hat{s}_i(p^{(k)})$ is resorted operation, in which the entries of $\hat{s}_i$ are rearranged in the same order of the permutation vector $p^{(k)}$.

Since the proposed SQRD-GGDex works as a SIC detector, the SQRD-GGDex is expected to outperform the ZF-GGDex. However, the performance of the SQRD-GGDex is strongly affected by the imperfect interference cancellation within each sub-system. It is worth emphasizing that the error propagation phenomenon in SQRD-GGDex detector can be lessened when all $l$ symbols in the $k$th sub-system correspond to $l$ strongest channel vectors. In other words, the $l$ symbols belonging to the first sub-system must be the strongest ones among a total of $N$ symbols, the next $l$ symbols are the strongest among the remaining $N-l$ ones and so on.

The probability, $Pr(A)$, that all $L$ sub-systems are detected with the least effect of error propagation can be determined as follows:

$$Pr(A) = \prod_{n=1}^{L} Pr(A_n) = \prod_{n=0}^{L-1} \left( \frac{1}{N-nl} \right)$$

(20)

One can easily see from (20) that $Pr(A)$ reduces as $L$ increases. This implies that the more sub-systems are generated, the more severe the error propagation effect becomes. As a consequence, the BER performance of SQRD-GGDex detector gets worse as the number of sub-systems gets larger.

**D. ZF-Presorted GG Dex and SQRD-Presort GGDex detectors**

As discussed above, the imperfect interference cancellation results in the performance degradation of the proposed detectors. Therefore, in this sub-section, a sort procedure is proposed to resolve the issue.

Based on Equation (5) the so-called total SNR for $k$th sub-system (denoted by $TSNR^{(k)}$) can be computed by the following equation:

$$TSNR^{(k)} = \frac{trace\left( E\left[ \left( P^{(k)^H} G_s s_k \right) \left( P^{(k)} G_s s_k \right)^H \right] \right)}{trace\left( E\left[ \left( P^{(k)^H} n_{ex}^{(k)} \right) \left( P^{(k)} n_{ex}^{(k)} \right)^H \right] \right)} = \frac{(IE_x) \|G_s^H P^{(k)}\|_F^2}{\|P^{(k)}\|_F^2} \leq (IE_x) \|G_s^H\|_F^2$$

(21)
Equation (21) implies that the Frobenius norm of the matrix $G_k$ plays an important role in the upper bound of the $TSNR^{(k)}$. A small value of the norm definitely lowers the $TSNR^{(k)}$ of the $k$th sub-system, thereby reducing the system performance. Besides, among the $L$ sub-systems, the reliability in the signal detection of the first ones is the most critical factor to eliminate the error propagation. These observations allow us to come up with a sort procedure applied to the extended channel matrix $U_{ex}$. In the procedure, the columns of $U_{ex}$ are sorted in such a way that their Frobenius norms are in a descending order, i.e., the first column has the largest norm, followed by the second, the third, and so on.

Mathematically, the columns of $U_{ex}$ are sorted to generate the pre-sorted channel matrix $U_{ex,s}$ and the associated permutation vector $q$ as:

$$
\left[ U_{ex,s} , q \right] = \text{Sort} \left( U_{ex} \right)
$$

(22)

where $\text{Sort}()$ denotes sort procedure; the sorted channel matrix $U_{ex,s}$ has the columns $u_{ex,s}^j$, $j=1,2,\ldots,N$, that satisfy $\|u_{ex,s}^1\|_F \geq \|u_{ex,s}^2\|_F \geq \cdots \geq \|u_{ex,s}^N\|_F$.

After the channel matrix $U_{ex,s}$ is obtained, the ZF-GGDex or the SQRD-GGDex can be used to estimate the transmitted signal vector $\hat{x}$. Finally, the re-order operation is required to get recovered signal vector as follows:

$$
\hat{x} = \hat{x}(q)
$$

(23)

Shown in Fig. 2 are the curves of the average $TSNR^{(1)}$ as functions of the SNR $p_{ul} / \sigma^2$ for the unsorted and sorted channels of the first sub-system among $L=2, 4, 8$ sub-systems. One can see from Fig. 2 that as the number of sub-systems increase, the total SNR reduces noticeably, particularly when the channel is unsorted. The results also demonstrate clearly that the sorted procedure allows the first sub-system to improve its total SNR remarkably as compared to the unsorted case. As a result, it is expected that the ZF-GGDex and SQRD-GGDex with the sort procedure can improve the system performance significantly.

4. Complexity analysis and BER performance comparison

A. Complexity analysis

In this sub-section, we evaluate the complexities of the aforementioned classical detectors, including the ZF, the MMSE, the SQRD, and the BLAST detectors, as well as the proposed ones. The complexities are evaluated by counting the number of floating
point operation (flops) required for the estimation of a transmitted vector at the BS. It is assumed that each real algebraic operation such as a real multiplication, a real division, a real addition, a real subtraction or square root of real number, is count as a flop [2], [8]. A complex multiplication and a complex division respectively require 6 and 11 flops.

Since the GGDex algorithm is applied to the extended system, where the channel matrix, the noise vector and the received vector include zero entries, the real as well as the complex entries. Hence, they should be taken into account for accurate evaluation of the complexities. Let us set \( a = N - kl \) and \( \mathbf{G}^{(k)} \) and \( \mathbf{G}_k \) as:

\[
\mathbf{G}^{(k)} = \begin{bmatrix} \mathbf{A}^{(k)T} & \mathbf{0}^{(k)T} & \mathbf{D}^{(k)T} \end{bmatrix}^T ; \mathbf{G}_k = \begin{bmatrix} \mathbf{A}_k^T & \mathbf{0}_k^T & \mathbf{D}_k^T & \mathbf{0}_{ik}^T \end{bmatrix}^T
\]

where \( \mathbf{A}^{(k)} \) and \( \mathbf{A}_k \) are respectively the \((N_r, \times a)\) and \((N_r, \times l)\) complex sub-matrices; \( \mathbf{D}^{(k)} \) and \( \mathbf{D}_k \) illustrate the \((a \times a)\) and \((l \times l)\) diagonal sub-matrices where all their main diagonal entries are real; \( \mathbf{0}^{(k)}, \mathbf{0}_k \) and \( \mathbf{0}_{ik} \) are respectively \((kl \times a)\), \(((k-1)l \times l)\) and \(((N-kl) \times l)\) zero sub-matrices.

**Tab. 1. Complexity of detectors in term of flops**

<table>
<thead>
<tr>
<th>Detector</th>
<th>Number of flops per estimated vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZF</td>
<td>(8N^3 + 16N^2N_r - 2N^2 + 6NN_r - 2N)</td>
</tr>
<tr>
<td>MMSE</td>
<td>(8N^3 + 16N^2N_r - 2N^2 + 6NN_r)</td>
</tr>
<tr>
<td>SQRD</td>
<td>(6N^2N_r + 5N^2 + 12NN_r + 3N)</td>
</tr>
<tr>
<td>BLAST</td>
<td>(\frac{15}{4}N^4 + 2N^3N_r + N^2N_r^2 + N(16N_r - 2))</td>
</tr>
<tr>
<td>ZF-GGDex</td>
<td>((L-1)(8blN_r + 2bi + 8b^2 + 8iN_r - 2N_r + 2IN + 2b - 2N_r + 8i^2 + 16i^2b - 2i^2 + 6i^2b - 2i^2 + 6i^2b - 2i^2)) + (\sum_{l=1}^{L-1}(24a^2N_r + 2a + 8a^3 - 4aN_r + 8N_rNa + 8N_r, kla - 2N_rkl + 2an + 2akl + 2a^2))</td>
</tr>
<tr>
<td>SQRD-GGDex</td>
<td>(L(6i^2b + 5i^2 + 12ib + 3l) + (L-1)(8blN_r + 2bi + 8b^2 + 8iN_r - 2N_r + 2IN + 2b - 2N_r + 8i^2 + 16i^2b - 2i^2 + 6i^2b - 2i^2 + 6i^2b - 2i^2 + 6i^2b - 2i^2)) + (\sum_{l=1}^{L-1}(24a^2N_r + 2a + 8a^3 - 4aN_r + 8N_rNa + 8N_r, kla - 2N_rkl + 2an + 2akl + 2a^2))</td>
</tr>
<tr>
<td>ZF-PreSorted GGDex</td>
<td>(C_{ZF-GGDex} + \frac{1}{2}(N^2 + 16bN - 7N))</td>
</tr>
<tr>
<td>SQRD-PreSorted GGDex</td>
<td>(C_{SQRD-GGDex} + \frac{1}{2}(N^2 + 16bN - 7N))</td>
</tr>
</tbody>
</table>

*Notes: \( N = KN_r; a = N - kl; b = N + N_r; l = mN_r \)
Under the above assumptions, the complexities of all considered detectors are computed and summarized in Tab. 1. It can be seen from Tab. 1 that the complexities of the proposed detectors are proportional to the 3rd order of $N$, which is the same complexity order as those of the classical ZF and the MMSE detectors.

For the sake of illustration, the complexities as functions of $L$ and of the system antenna configurations are depicted in Fig. 3 and Fig. 4 for various detectors. Fig. 3 shows the complexities of the classical ZF, the MMSE, the SQRD and the BLAST detectors as well as those of the proposed ones when the system antenna configuration is fixed at $N_r = 64$, $K = 16$, $N_c = 4$ and $L$ is changed in the range of $L = [2, 16]$.

It can be seen from Fig.4 that the complexities of proposed detectors are higher than those of classical ZF, the MMSE and the SQRD detectors while much lower than that of the BLAST for all values of $L$. The results also show that the computational costs of the ZF-GGDex, The SQRD-GGDex, the ZF-Presorted GGDex and the SQRD-Presorted GGDex detectors are almost the same. In addition, the proposed detectors have the lowest detection complexities when $L = 2$. The larger values of $L$ require more matrix inversions to be carried out, leading to higher detection complexities. Note that the higher complexities of proposed detectors when $L > 2$ come at prices of their SNR gains as illustrated in the next sub-section. Undoubtedly, the higher complexities are compensated by noticeable improvements in the system performance as illustrated by the simulation results below.

![Fig. 3. Complexity vs. number of sub-system](image)

![Fig. 4. Complexity vs. number of antennas](image)
The complexities of the MMSE, the SQRD, the BLAST and the SQRD-Presorted GGDex detectors versus $N_r$ at the BS is depicted in Fig. 5. We consider the scenarios that $N_r = N$ changes in the range of $[60, 200]$ for $L = 2, 4, 8$. We can see from Fig. 5 that the complexities of all the detectors increase when $N_r$ increases. Particularly, there is a sharp increase in the complexity of the BLAST as $N_r$ increases. The gaps between the complexity curves of proposed detectors and the ones of the classical SQRD and the MMSE detectors get larger as $N_r$ and/or $L$ increases. Nevertheless, they become marginal as the number of sub-systems is equal to $L = 2$.

B. BER performance comparison

In this sub-section, BER performances of the proposed detectors are compared to those of the classical ZF, the MMSE, the BLAST and the SQRD ones. It is assumed that $N_r = 64$, $K = 16$, and $N_r = 4$. Besides, 4-QAM modulation technique is employed. The cell radius $r$ and reference distance $d_0$ are set equal to 1000 meters and 100 meters, respectively. In the simulations, all $K$ users are randomly distributed in the cell where the distance from $k$th user to the BS is chosen randomly in the range of $[200, 990]$ meters. The deviation of the log-normal shadowing fading coefficient is $\sigma_{\text{shadow}} = 8$ dB and the path loss component is $\gamma = 3.5$. The channel is assumed to be quasi-static, i.e., it remains unchanged within 200 symbol periods and changes independently from one realization to the next. The noise term is assumed to be i.i.d random variable with zero mean and unit variance. In addition, the transmit powers of all active users are assumed to be the same. The BER curves are drawn as functions of the SNR, defined as $(p_{ul}/\sigma^2)$.
Shown in Fig. 5 are the BER curves of the proposed ZF-GGDex and the ZF-Presorted GGDex detectors when $L = 2, 4, 8$ sub-systems are generated. Besides, the BER curves for the conventional ZF, the MMSE and the BLAST detectors are provided. It can be seen from Fig. 5 that the ZF-GGDex detector offers remarkable performance improvement as compared to both the ZF and the MMSE ones. As the pre-sort procedure is applied, the ZF-Presorted GGDex provides more significant performance improvement. The more sub-systems are generated, the higher performance both the ZF-GGDex and the ZF-Presorted GGDex detectors can achieve. Specifically, compared to the classical MMSE, the ZF-GGDex achieves SNR gains of about 2.2, 4.4 and 6.2 dB at $BER = 10^{-4}$ and $L = 2, 4, 8$, respectively. Under the same conditions, the SNR gains offered by the ZF-Presorted GGDex increase up to around 15, 18 and 19 dB correspondingly for $L = 2, 4, 8$. More importantly, when $L = 8$, performance of the ZF-Presorted GGDex becomes comparable to that of the BLAST while the ZF-Presorted GGDex enables the system to remarkably reduce the detection complexity (see Fig. 4).

Figure 6 illustrates the BER curves of the proposed SQRD-GGDex and the SQRD-Presorted GGDex detectors when $L = 2, 4, 8$ in comparison with those of the conventional ZF, the MMSE and the BLAST ones. One can observe from Fig. 6 that the SQRD-GGDex detector suffers from BER degradation as $L$ gets larger. The simulation results confirm our qualitative analysis in Sub-section 3.D regarding the error propagation effect as $L$ increases. For example, at $BER = 10^{-4}$ and $L = 2$, the SQRD-GGDex provides an SNR gain of 16.3 dB as compared to the MMSE. In contrast, the gain reduces to around 11.6 dB as $L = 8$, a noticeable degradation in the system performance. Fortunately, by applying proposed pre-sorted procedure, this phenomenon can be completely removed. The BER curves of the SQRD-Presorted GGDex detector are nearly identical to that of the BLAST regardless of $L$.

It is worth emphasizing that both the SQRD-GGDex and the SQRD-Presorted GGDex detectors have the lowest complexities when $L = 2$. Therefore, for these two detectors, the optimum number of sub-systems to be generated is 2.

5. Conclusion

In this paper, we have proposed a Generalized Group Detection algorithm for signal recovery in Massive MIMO systems, called GGDex. The GGDex algorithm
allows us to build two efficient detectors, called the ZF-GGDex and the SQRD-GGDex. Although the two detectors significantly improve system performance as compared to their conventional MMSE counterpart, they severely suffer from the error propagation phenomenon, thereby degrading their performances. In order to further improve the quality of these detectors, we have proposed two other detectors called the ZF-Presorted GGDex and the SQRD-Presorted GGDex ones by applying a sort procedure to the extended channel matrix prior to the adoption of either the ZF-GGDex or the SQRD-GGDex. Simulation results show that the ZF-Presorted GGDex and the SQRD-Presorted GGDex also remarkably outperform BER their MMSE counterpart. Performances of the ZF-Presorted GGDex and SQRD-Presorted GGDex even approach that of the classical BLAST detector at noticeably lower detection complexity. Therefore, the proposed detectors are potential candidates for signal recovery at the base stations of Massive MIMO systems with respect to both performance and detection complexities.

References

TÁCH TÌNH HIỆU THEO NHÓM TRONG CÁC HỆ THỐNG MASSIVE MIMO

Tóm tắt: Trong bài báo này, chúng tôi đề xuất thuật toán tách sóng theo nhóm (gọi tắt là GGDex) để khôi phục tín hiệu phát trong các hệ thống Massive MIMO. Thuật toán được xây dựng bằng cách chia hệ thống Massive MIMO mở rộng tương đương thành một số hệ thống con tùy ý, trong đó các tín hiệu phát được tách theo phương pháp triệt nhiễu nối tiếp SIC. Tiếp đó, chúng tôi đề xuất bốn bộ tách sóng mới được đặt tên là ZF-GGDex, SQRD-GGDex, ZF-Presorted GGDex và SQRD-Presorted GGDex. Kết quả phân tích và mô phỏng cho thấy các bộ tách tín hiệu được đề xuất cải thiện phẩm chất lỗi bít khoảng hơn 3 dB so với các bộ tách tuyến tính truyền thông. Đặc biệt là, phẩm chất lỗi của các bộ tách ZF-Presorted GGDex và SQRD-Presorted GGDex tiệm cận phẩm chất của bộ tách sóng BLAST. Chính vì thế, các bộ tách được đề xuất trong bài báo này phù hợp để ứng dụng trong các hệ thống Massive MIMO.

Từ khóa: Massive MIMO; đường lên; máy thu có độ phức tạp thấp; tách tín hiệu theo nhóm.

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